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TWO LEVELS OF STORAGE MODELS FOR DETERIORATING ITEMS WITH STOCK DEPENDENT DEMAND AND SHORTAGES

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ABSTRACT

In this paper an inventory model for deteriorating products with two warehouse facility has been developed. The first warehouse is the owned warehouse and the second one is the rented warehouse. The deterioration rate in O.W. is a linear function of time and due to different preservation facility and storage environment the deterioration rate in R.W. is assumed to be negligible and due to the same reason the holding cost in R.W. is higher than the holding cost in owned warehouse. The Demand rate for the product is considered to be a stock dependent function. The stock transferred from rented warehouse to own warehouse follows a bulk release rule. Different examples with different demand rate, warehouse capacity and deterioration rate are also illustrated in the model. In every case the optimal value of T.C., sales revenue and average net profit has been found. With different examples it is observed that the model is quite stable for different value of system parameters. The results presented in this model are also interpreted graphically.

Keywords: Two warehouse; Deterioration; Stock Dependent demand; Inventory

1. INTRODUCTION

The amount of inventory displayed in the showrooms play an important role towards the demand of the products. A high stock level attracts the customers to buy more. So the managers of a company are compelled to maintain the stock level and sometimes due to offered concessions in bulk purchasing the companies are forced to buy more than their needs. The classical inventory models are developed under the assumption that the owned warehouse or rented warehouse has unlimited capacity. In this case the companies need an extra warehouse to stock the inventory. This may be a rented warehouse or a new rebuild warehouse. From economic part of view generally it is observed that to choose a rented warehouse is a better option. Traditionally many researchers worked on two warehouse inventory system. Hartely (1976) was the first to introduce such type of models in inventory. This model was developed under the assumption that the holding cost in rented warehouse is greater than the holding cost in owned warehouse. Sharma (1987) developed a two warehouse deterministic inventory model for deteriorating products with shortages and constant demand. Pakkala and Achary (1992) extended this two warehouse inventory model for deteriorating items and finite replenishment rate. Beside these Benkherouf (1997) and Bhunia and Maiti (1998) introduced the idea of time dependent demand rate in two warehouse inventory system. Yang (2004) introduced a two storage inventory model for a single items and shortages. Hsieh *et al.* (2008) presented an inventory model for deteriorating products with two warehouses and minimized the net present value of the total cost. Arya et al. (2009) developed an order level inventory model for perishable items with stock dependent demand and partial backlogging. Singh and Singh (2013) introduced an optimal ordering policy for deteriorating items with power▾form stock dependent demand under two▾warehouse storage facility. Tayal et al. (2014) developed a deteriorating production inventory problem with space restriction. In this model the extra ordered quantity is returned to the supplier for which the supplier charges a penalty cost on the retailer. Stock level is also an important factor that affects the demand of a product. Recently many researchers worked on stock dependent demand related inventory models. Khurana et al. (2015) came forward with a supply chain production inventory model for deteriorating product with stock dependent demand under inflationary environment and partial backlogging. Singh et al. (2016) introduced an inventory model for deteriorating items having seasonal and stock-dependent demand with allowable shortages. Tayal et al. (2016) introduced an integrated production inventory model for perishable products with trade credit period and investment in preservation technology. In this model the demand rate is considered as price sensitive function. Singh et al. (2016) also presented an economic order quantity model for deteriorating products having stock dependent demand with trade credit period and preservation technology. Pramanik et al (2017) developed an integrated supply chain model under three level trade credit policy with price, credit period and credit amount dependent demand, where a supplier offers a credit period to his/her wholesaler to boost the demand of the item.

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In addition due to the deterioration of the products, to consider the effect of deterioration is vital in practical environment. Ghare and Schrader (1963) were the first to introduce the concept of deterioration in inventory modelling. Later Sharma (1987) developed a two warehouse inventory model for deteriorating products with constant rate of deterioration. Pakkala and Achary (1994) introduced a two warehouse inventory model for deteriorating items with bulk release pattern. Yang (2004) considered a two warehouse inventory model for deteriorating items with complete backlogging of occurring shortages. In most of the models the deterioration rate is assumed as a constant. Singh et al. (2009) presented an EOQ model for perishable items with power demand pattern and partial backlogging. Tayal et al. (2014) developed a two echelon supply chain model for deteriorating items with effective investment in preservation technology. In this model the rate of deterioration is taken as time dependent and a preservation technology cost is applied to reduce the existing rate of deterioration. Singh et al. (2014) presented a multi item inventory model for deteriorating items with expiration date and allowable shortages. Tayal et al. (2015) considered an inventory model for deteriorating items with seasonal products and an option of an alternative market. However there are so many products which maintain its quality for a fix period of time and after that it begins to deteriorate. Later Tayal et al. (2015) developed an EPQ model for non-instantaneous deteriorating item with time dependent holding cost and exponential demand rate.

In this paper, optimization framework has been presented to derive the optimal replenishment policy in a two warehouse inventory system for deteriorating products to minimize the total cost and maximize the net profit. A numerical example and sensitivity analysis are also presented to illustrate the model. From sensitivity analysis it is observed that with the increment in demand parameter total cost and net profit of the system increases, while in the case of deterioration rate with the increment in deterioration rate the net profit of the system decreases.

2. ASSUMPTIONS AND NOTATIONS

Assumptions:

1. The products assumed in this model are deteriorating in nature.
2. Demand rate is a function of on hand inventory and is given by $D(t)=a+bI_0(t)$
3. This is a two warehouse inventory model.
4. For owned warehouse deterioration rate is assumed to be a linear function of time.
5. Deterioration rate in rented warehouse is assumed to be negligible due to high preservation facilities.
6. The deteriorated units will not be replaced or repaired.
7. The O.W. has a fix capacity of W units and the rented warehouse has the capacity of R units. If the amount of inventory is greater than W+R, then the company has to pay for any other warehouse.
8. The holding cost in R.W. is greater than the holding cost in O.W.
9. The transfer of inventory from rented warehouse to owned warehouse follows the bulk release rule.
10. The replenishment rate is infinite.

Notataions

a,b	demand parameter
θ	deterioration parameter
W	owned warehouse capacity
A	set up cost per replenishment cycle
S	total purchased inventory
c	unit purchasing cost
h_1	holding cost per unit in O.W.
h_2	holding cost per unit in R.W.
n	total number of cycles
p	selling price per unit

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$I_0(t)$	inventory level at any time t in O.W.
$I_r(t)$	inventory level at any time t in R.W.
s	transportation cost per shipment
K	the amount of stock transferred in single shipment from R.W. to O.W.

3. MATHEMATICAL FORMULATION AND SOLUTION OF THE MODEL

The model is developed under the assumption that a company purchases S (S>W and S≤W+R) units. Out of these S units W units are kept in O.W. and remaining (S-W) units are kept in R.W. At start, the occurring demand is satisfied from O.W. until the stock level drops to W/4 units. At this time $t=T_1$ the k units are transferred from R.W. to O.W. This process is carry on until the stock in R.W is finished. This inventory system is shown in below mentioned fig. 1 and fig. 2.

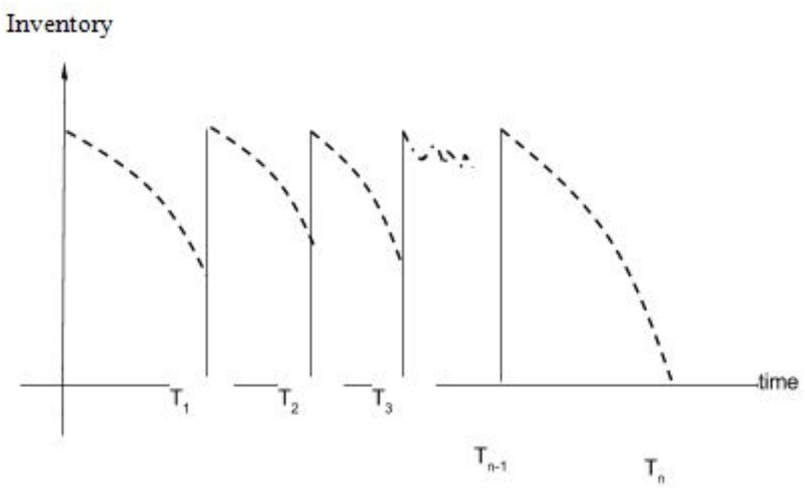

Fig. 1 Owned warehouse inventory system

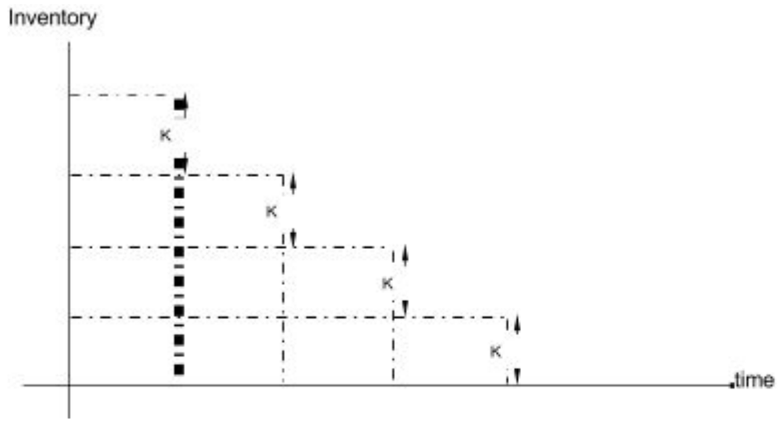

Fig2. Rented warehouse inventory system

The differential equation showing the inventory time behavior of the system in owned warehouse:

$$\frac{dI_0(t)}{dt} = -\theta t I_0(t) - (a + bI_0(t)) \qquad T_i \le t \le T_{i+1} \qquad (1)$$

with boundary condition:

$$I_0(T_i) = W \qquad \text{for } i=0,1,2\ldots\ldots(n-1) \qquad (2)$$

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Here $T_0 = 0$ and $T_n = T$

The solution of this equation is given as follow:

$$I_0(t) = \{a(T_i - t) + \frac{b}{2}(T_i^2 - t^2) + \frac{\theta}{6}(T_i^3 - t^3)\}e^{-(bt+\frac{\theta t^2}{2})} + We^{\{b(T_i-t)+\frac{\theta}{2}(T_i^2-t^2)\}}$$

$$T_i \le t \le T_{i+1} \quad (3)$$

The equation showing the inventory time behavior of the system in rented warehouse:

$$I_r(t) = R \qquad 0 \le t \le T_1 \quad (4)$$

$$I_r(T_{i+1}) = I_r(T_i) - K \qquad T_i \le t \le T_{i+1} \quad (5)$$

for i=1,2,3…………(n-2)

Different Associated Cost:

(1) Set up cost:

The order is placed at the beginning of every replenishment cycle. So the set up cost for every replenishment cycle will be:

S.P.C.=A (6)

(2) Purchasing Cost:

The S units of inventory are bought at the beginning of each cycle, so the total purchasing cost will be:

P.C.=SC (7)

(3) Holding Cost:

The inventory exists during $T_i \le t \le T_{i+1}$, i=0,1,2………….(n-1). Hence the holding cost will be:

Holding cost in Owned Warehouse:

$$H.C_{O.W} = \sum_{i=0}^{n-1} h_1 \int_{T_i}^{T_{i+1}} I_0(t)dt$$

$$H.C_{O.W} = \sum_{i=0}^{n-1} h_1 [\{a(T_iT_{i+1} - \frac{T_{i+1}{}^2}{2}) + \frac{b}{2}(T_i^2T_{i+1} - \frac{T_{i+1}{}^3}{3}) + \frac{\theta}{6}(T_i^3T_{i+1} - \frac{T_{i+1}{}^4}{4}) - ab(T_i\frac{T_{i+1}{}^2}{2} - \frac{T_{i+1}{}^3}{3}) - \frac{\theta a}{2}(T_i\frac{T_{i+1}{}^3}{3} - \frac{T_{i+1}{}^4}{4})\}$$

$$+W\{T_{i+1} + b(T_iT_{i+1} - \frac{T_{i+1}{}^2}{2}) + \frac{\theta}{2}(T_i^2T_{i+1} - \frac{T_{i+1}{}^3}{3})\} - \{a\frac{T_i^2}{2} + \frac{b}{3}T_i^3 + \frac{\theta}{8}T_i^4 - \frac{ab}{6}T_i^3 - \frac{\theta a}{24}T_i^4\}]$$

$$-W\{T_i + \frac{b}{2}T_i^2 + \frac{\theta}{3}T_i^3\}] \quad (8)$$

Holding cost in Rented Warehouse:

$$H.C._{R.W.} = h_2\frac{(S-W)}{2}T_{n-1} \quad (9)$$

(4) Transportation Cost

Inventory is transferred from the R.W. to O.W. in (n-2) shipments, therefore the associated cost will be:

$$T.R.C. = s(n-2) \quad (10)$$

(5) Sales Revenue

Inventory is used to satisfy the demand during $T_i \le t \le T_{i+1}$, i=0,1,2………(n-1). So the total sales revenue earned during this time:

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$$S.R. = \sum_{i=0}^{n-1} p \int_{T_i}^{T_{i+1}} D(t)\,dt$$

$$S.R. = \sum_{i=0}^{n-1} p\{(a+Wb)(T_{i+1}-T_i) + ba(T_i T_{i+1} - \frac{T_{i+1}{}^2}{2} - \frac{T_i^2}{2})\} \qquad (11)$$

(6) Total Profit

Inventory is used to satisfy the demand during $T_i \le t \le T_{i+1}$, i=0,1,2………(n-1). So the total profit earned during this time:

Profit = S.R. – P.C. – $H.C_{O.W.}$ – $H.C_{R.W.}$ – T.R.C. – S.P.C. (12)

4. NUMERICAL EXAMPLE

a=100 units, b=0.05, W=500 units, h_1=0.5 rs/unit, h_2=0.6 rs/unit, θ=0.001, c=12 rs/unit, p=20 rs/unit, T_0=0, T_n= T

By taking these system parameters the above model is analyzed for different integral values of n=1,2….6 and different solutions are found which are listed in below mentioned table.

Example 1: a=100 units, W=500 units, θ=0.001

Table1(a): Analysis for different number of cycles:

n	T_1	T_2	T_3	T_4	T_5	T_6
1	28.2903	-	-	-	-	-
2	28.2903	43.1692	-	-	-	-
3	28.2903	43.1692	54.0864	-	-	-
4	28.2903	43.1692	54.0864	62.9463	-	-
5	28.2903	43.1692	54.0864	62.9463	70.5104	-
6	28.2903	43.1692	54.0864	62.9463	70.5104	77.1698

Table 1 (b): Optimal values for different number of cycles:

n	S	T.C.	N.P.	Revenue	N.P./T
1	500	10282	20426.2	30708.2	722.021
2	600	22837.2	33999.5	56836.7	787.586
3	700	35400.9	42769.8	78170.7	790.768
4	800	47515.3	48880.2	96395.5	776.538
5	900	59229.2	53215.8	112445	754.722
6	1000	70628.6	56247.5	126875.1	728.879

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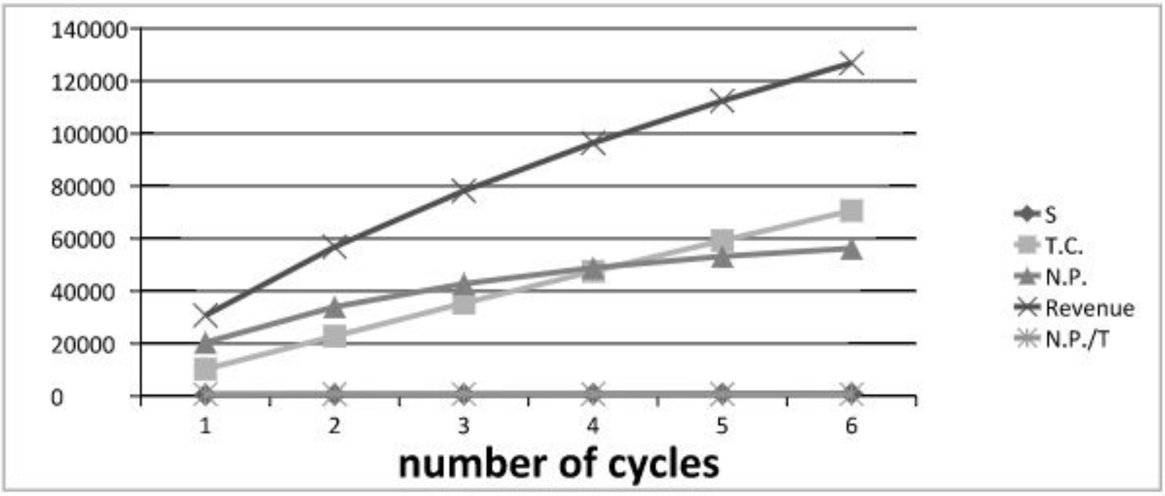

fig. 3: Variation of Total Cost, Revenue, Net Profit and Average Profit with number of cycles (Example 1)

From this analysis it is observed from table 1 (a) and table1(b) that as the number of cycle increases, the net profit of the system also increases, but the net profit per unit time is found to be maximum at n=3. This variation in system parameters has been shown in above mentioned fig. 3.

Example 2: a=200 units, W=500 units, θ=0.001

Table 2 (a): Analysis for different number of cycles:

n	T_1	T_2	T_3	T_4	T_5	T_6
1	25.9684	-	-	-	-	-
2	25.9684	40.0996	-	-	-	-
3	25.9684	40.0996	50.5686	-	-	-
4	25.9684	40.0996	50.5686	59.1062	-	-
5	25.9684	40.0996	50.5686	59.1062	66.4167	-
6	25.9684	40.0996	50.5686	59.1062	66.4167	72.8658

Table 2 (b): Optimal solution for different number of cycles:

n	S	T.C.	N.P.	Revenue	N.P./T
1	500	9018.5	40403.5	49422	1555.87
2	600	26506.5	66536.8	93043.3	1659.2
3	700	44192	85001.8	129193.8	1680.92
4	800	61137.5	99186.4	160323.9	1678.104
5	900	77344.4	110533	187877.4	1664.23
6	1000	92925.3	119813	212738.3	1644.296

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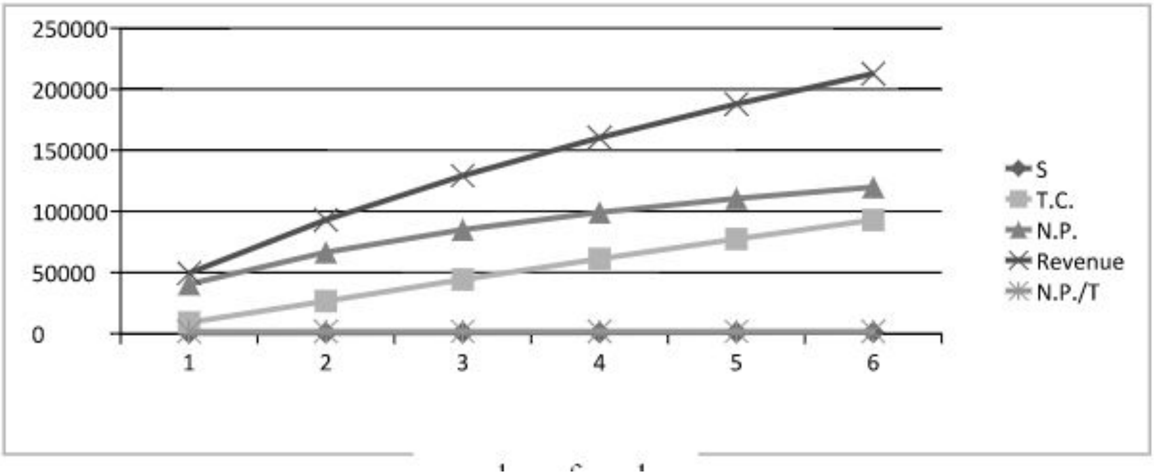

fig. 4: Variation of Total Cost, Revenue, Net Profit and Average Profit with number of cycles(Example 2)

From this analysis it is observed that as the number of cycle increases the net profit of the system increases, but it is optimal for n=3. It is also observed from table 2 (a) and table 2 (b) that with the increased demand the net profit and sales revenue of the system are also increased. This variation is depicted in figure 4.

Example 3: a=200 units, W=750 units, θ=0.001

Table 3 (a): Analysis for different number of cycles:

n	T_1	T_2	T_3	T_4	T_5	T_6
1	30.6726	-	-	-	-	-
2	30.6726	46.2565	-	-	-	-
3	30.6726	46.2565	57.5837	-	-	-
4	30.6726	46.2565	57.5837	66.734	-	-
5	30.6726	46.2565	57.5837	66.734	74.5246	-
6	30.6726	46.2565	57.5837	66.734	74.5246	81.3707

Table 3 (b): Optimal solution for different number of cycles:

n	S	T.C.	N.P.	Revenue	N.P./T
1	500	16199.1	21110.1	37309.2	688.239
2	600	31758.2	36264	68022.2	783.976
3	700	46711.3	46045.5	92756.8	799.631
4	800	60832.4	52901.3	113733.7	792.718
5	900	74299.5	57823.6	132123.1	775.899
6	1000	87271.4	61335.1	148606.5	753.773

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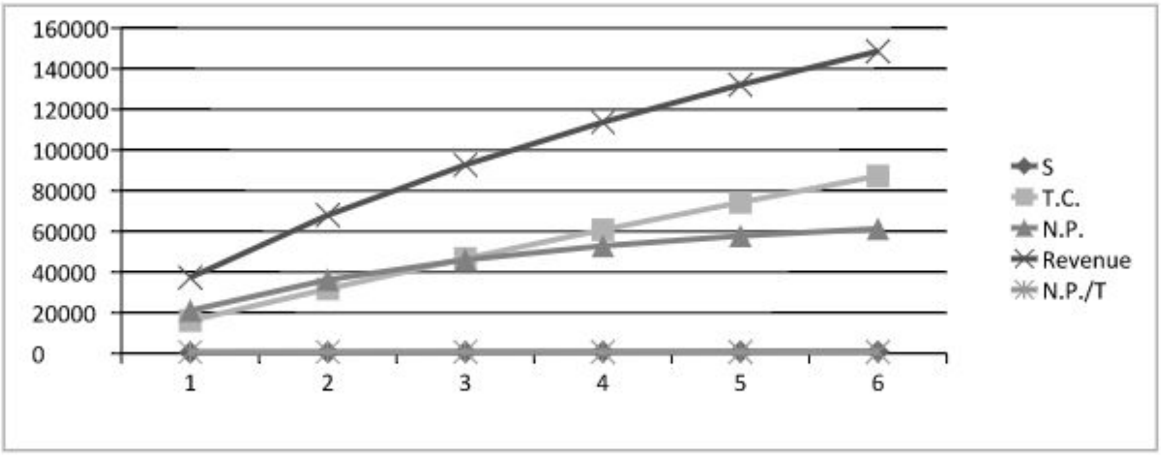

Fig. 5: Variation of Total Cost, Revenue, Net Profit and Average Profit with number of cycles (Example 3)

It is observed from this analysis that with the increased number of cycles the net profit of the system also increases, but the net profit per unit time of the system is maximum for n=3 and after that it decreases. It is also observed from these tables 3(a) and 3(b) that with the increment in owned warehouse capacity, the T.C. and net profit of the system also increases. This variation is depicted in figure 5.

Example 4: : a=200 units, W=750 units, θ=0.002

Table 4 (a): Analysis for different number of cycles:

n	T_1	T_2	T_3	T_4	T_5	T_6
1	26.9873	-	-	-	-	-
2	26.9873	40.5975	-	-	-	-
3	26.9873	40.5975	50.2726	-	-	-
4	26.9873	40.5975	50.2726	57.9809	-	-
5	26.9873	40.5975	50.2726	57.9809	64.4819	-
6	26.9873	40.5975	50.2726	57.9809	64.4819	70.1553

Table 4 (b): Optimal solution for different number of cycles:

n	S	T.C.	N.P.	Revenue	N.P./T
1	500	11654.8	19397.7	31052.5	718.771
2	600	24218.5	31597.8	55816.3	778.318
3	700	36213.5	39110.1	75323.6	777.96
4	800	47488.2	44135.4	91623.6	761.205
5	900	58217.7	47545.2	105762.9	737.341
6	1000	68546.3	49790.7	118337	709.721

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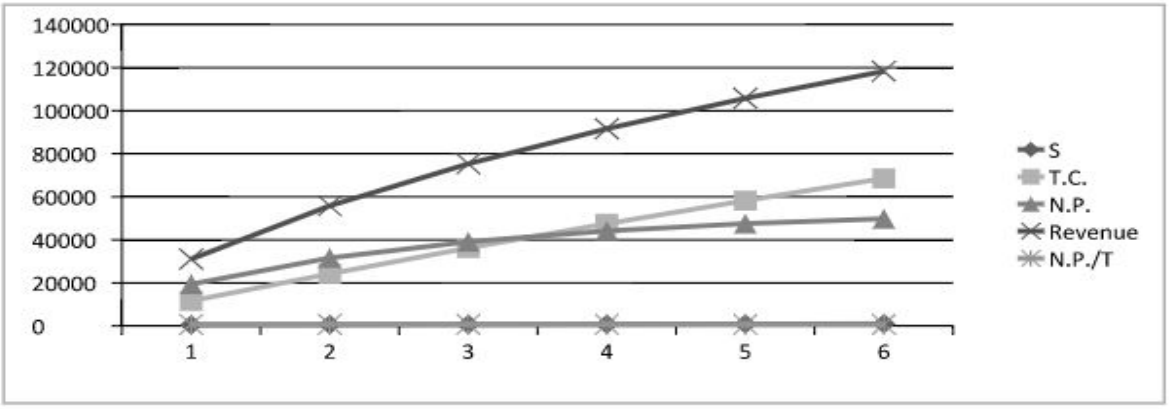

Fig. 6: Variation of Total Cost, Revenue, Net Profit and Average Profit with number of cycles(Example 4)

From these tables 4 (a) and 4 (b) we observe that as the value of n increases the N.P. of the system also increases. The net profit per unit time is maximum for n=2. From this we also observe that with the increased rate of deterioration the net profit of the system is decreased and it can also be seen in figure 6.

From this analysis it is observed that as the value of demand parameter 'a' increases, the total cost and net profit of the system increases and the average net profit is maximum for n=3. In the case of increases capacity of owned warehouse, the total cost and net profit of the system increases rapidly, but the average net profit shows a slight change in its value and is maximum for n=3. Further with the increment in deterioration parameter, the total cost increases but net profit decreases and it is observed that average net profit is decreases but maximum is at n=3.

5. CONCLUSION

In this paper an inventory model for deteriorating items with two storage facility has been developed. The demand for the products is assumed to be a function of stock level. The time dependent rate of deterioration is considered for owned warehouse and due to high preservation facility the deterioration in rented warehouse is assumed to be negligible. The holding cost in owned warehouse is smaller than the rented warehouse due to high storage facility. The bulk release pattern is used to transfer the stock from rented warehouse to owned warehouse. The model is analyzed numerically up to six cycles for different system parameters and the optimal solution is found among these. The average net profit is found to be maximum for n=3, which shows that the model presented here is quite stable.

The model can be extended further for price dependent demand, quantity discount, time value of money and more generalized pattern for deterioration rate.

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7. NOTES ON CONTRIBUTORS

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